Appendix A: and Hints to Selected Exercises

Appendix A: Answers and Hints to Selected Exercises

Answers and Hints to Selected Exercises

Chapter 1

Section 1.1 (p. § 1.1)

1. (a) $\sqrt{5}$ (b) $\sqrt{5}$ (c) $\sqrt{17}$ (d) 1
(e) $2\sqrt{17}$ **2.** Yes **3.** No

Section 1.2 (p. § 1.2)

1. (a) $(-4,4,-3)$ (b) $(2,6,-1)$
(c) $\left(\frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-2}{\sqrt{30}}\right)$ (d) $\frac{\sqrt{41}}{2}$ (e) $\frac{\sqrt{41}}{2}$
(f) $(14,-6,8)$ (g) $(-7,3,-4)$
(h) $(-1,-6,1)$ (i) $(-2,-4,2)$ (j) No.
3. No. $\|\mathbf{v}\| + \|\mathbf{w}\|$ is larger.

Section 1.3 (p. § 1.3)

1. 10 **3.** 73.4° **5.** 90° **7.** 0°
9. Yes, since $\mathbf{v} \cdot \mathbf{w} = 0$.
11. $|\mathbf{v} \cdot \mathbf{w}| = 0 < \sqrt{21}\sqrt{5} = \|\mathbf{v}\| \, \|\mathbf{w}\|$
13. $\|\mathbf{v} + \mathbf{w}\| = \sqrt{26} < \sqrt{21} + \sqrt{5} = \|\mathbf{v}\| + \|\mathbf{w}\|$
15. Hint: use Definition 1.6.
24. Hint: See Theorem 1.10(c).

Section 1.4 (p. § 1.4)

1. $(-5,-23,-24)$ **3.** $(8,4,-5)$ **5.** $\mathbf{0}$
7. 16.72 **9.** $4\sqrt{5}$ **11.** 9 **13.** 0
and $(8,-10,2)$ **15.** 14

Section 1.5 (p. § 1.5)

1. (a) $(2,3,-2)+t(5,4,-3)$ (b) $x=2+5t$, $y=3+4t$, $z=-2-3t$ (c) $\frac{x-2}{5}=\frac{y-3}{4}=\frac{z+2}{-3}$
3. (a) $(2,1,3)+t(1,0,1)$ (b) $x=2+t$, $y=1$, $z=3+t$ (c) $x-2=z-3$, $y=1$
5. $x=1+2t$, $y=-2+7t$, $z=-3+8t$
7. 7.65 **9.** $(1,2,3)$
11. $4x-4y+3z-10=0$
13. $x-2y-z+2=0$
15. $11x-24y+21z-26=0$ **17.** $9/\sqrt{35}$
19. $x=5t$, $y=2+3t$, $z=-7t$ **21.** $(10,-2,1)$

Section 1.6 (p. § 1.6)

1. radius: 1, center: $(2,3,5)$ **3.** radius: 5, center: $(-1,-1,-1)$ **5.** No intersection.
7. circle $x^2+y^2=4$ in the planes $z=\pm\sqrt{5}$
9. lines $\frac{x}{a}=\frac{y}{b}$, $z=0$ and $\frac{x}{a}=-\frac{y}{b}$, $z=0$
13. $\left(\frac{2a}{2-c},\frac{2b}{2-c},0\right)$

Section 1.7 (p. § 1.7)

1. (a) $(4,\frac{\pi}{3},-1)$ (b) $(\sqrt{17},\frac{\pi}{3},1.816)$
3. (a) $(2\sqrt{7},\frac{11\pi}{6},0)$ (b) $(2\sqrt{7},\frac{11\pi}{6},\frac{\pi}{2})$
5. (a) $r^2+z^2=25$ (b) $\rho=5$
7. (a) $r^2+9z^2=36$ (b) $\rho^2(1+8\cos^2\phi)=36$
10. $(a,\theta,a\cot\phi)$ **12.** Hint: Use the distance formula for Cartesian coordinates.

Section 1.8 (p. § 1.8)

1. $\mathbf{f}\,'(t)=(1,2t,3t^2)$; $x=1+t$, $y=z=1$
3. $\mathbf{f}\,'(t)=(-2\sin 2t,2\cos 2t,1)$; $x=1$, $y=2t$, $z=t$ **5.** $\mathbf{v}(t)=(1,1-\cos t,\sin t)$, $\mathbf{a}(t)=(0,\sin t,\cos t)$
9. (a) Line parallel to **c** (b) Half-line parallel to **c** (c) Hint: Think of the functions as position vectors.
15. Hint: Theorem 1.16

Section 1.9 (p. § 1.9)

1. $\frac{3\pi\sqrt{5}}{2}$ **3.** $2(5^{3/2}-8)$ **5.** Replace t by $\left(\frac{s+16}{2}\right)^{2/3}-4$ **6.** Hint: Use Theorem 1.20(e), Example 1.37, and Theorem 1.16 **7.** Hint: Use Exercise 6.

9. Hint: Use $\mathbf{f}\,'(t) = \|\mathbf{f}(t)\|\mathbf{T}$, differentiate that to get $\mathbf{f}\,''(t)$, put those expressions into $\mathbf{f}\,'(t)\,\times\,\mathbf{f}\,''(t)$, then write $\mathbf{T}\,'(t)$ in terms of $\mathbf{N}(t)$. **11.** $\mathbf{T}(t) = \frac{1}{\sqrt{2}}(-\sin t, \cos t, 1)$, $\mathbf{N}(t) = (-\cos t, -\sin t, 0)$, $\mathbf{B}(t) = \frac{1}{\sqrt{2}}(\sin t, -\cos t, 1)$, $\kappa(t) = 1/2$

Chapter 2

Section 2.1 (p. § 2.1)

1. domain: $\mathbb{R}^2$, range: $[-1,\infty)$ **3.** domain: $\{(x,y) : x^2+y^2 \ge 4\}$, range: $[0,\infty)$
5. domain: $\mathbb{R}^3$, range: $[-1,1]$ **7.** 1
9. does not exist **11.** 2 **13.** 2 **15.** 0
17. does not exist

Section 2.2 (p. § 2.2)

1. $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 2y$ **3.** $\frac{\partial f}{\partial x} = x(x^2+y+4)^{-1/2}$, $\frac{\partial f}{\partial y} = \frac{1}{2}(x^2+y+4)^{-1/2}$ **5.** $\frac{\partial f}{\partial x} = ye^{xy}+y$, $\frac{\partial f}{\partial y} = xe^{xy}+x$ **7.** $\frac{\partial f}{\partial x} = 4x^3$, $\frac{\partial f}{\partial y} = 0$
9. $\frac{\partial f}{\partial x} = x(x^2+y^2)^{-1/2}$, $\frac{\partial f}{\partial y} = y(x^2+y^2)^{-1/2}$
11. $\frac{\partial f}{\partial x} = \frac{2x}{3}(x^2+y+4)^{-2/3}$, $\frac{\partial f}{\partial y} = \frac{1}{3}(x^2+y+4)^{-2/3}$ **13.** $\frac{\partial f}{\partial x} = -2xe^{-(x^2+y^2)}$, $\frac{\partial f}{\partial y} = -2ye^{-(x^2+y^2)}$ **15.** $\frac{\partial f}{\partial x} = y\cos(xy)$, $\frac{\partial f}{\partial y} = x\cos(xy)$ **17.** $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = 2$, $\frac{\partial^2 f}{\partial x\,\partial y} = 0$ **19.** $\frac{\partial^2 f}{\partial x^2} = (y+4)(x^2+y+4)^{-3/2}$, $\frac{\partial^2 f}{\partial y^2} = -\frac{1}{4}(x^2+y+4)^{-3/2}$, $\frac{\partial^2 f}{\partial x\,\partial y} = -\frac{1}{2}x(x^2+y+4)^{-3/2}$
21. $\frac{\partial^2 f}{\partial x^2} = y^2e^{xy}$, $\frac{\partial^2 f}{\partial y^2} = x^2e^{xy}$, $\frac{\partial^2 f}{\partial x\,\partial y} = (1+xy)e^{xy}+1$ **23.** $\frac{\partial^2 f}{\partial x^2} = 12x^2$, $\frac{\partial^2 f}{\partial y^2} = 0$, $\frac{\partial^2 f}{\partial x\,\partial y} = 0$ **25.** $\frac{\partial^2 f}{\partial x^2} = -x^{-2}$, $\frac{\partial^2 f}{\partial y^2} = -y^{-2}$, $\frac{\partial^2 f}{\partial x\,\partial y} = 0$

Section 2.3 (p. § 2.3)

1. $2x+3y-z-3=0$ **3.** $-2x+y-z-2=0$
5. $x+2y=z$ **7.** $\frac{1}{2}(x-1)+\frac{4}{9}(y-2)+\frac{\sqrt{11}}{12}(z-\frac{2\sqrt{11}}{3})=0$ **9.** $3x+4y-5z=0$

Section 2.4 (p. § 2.4)

1. $(2x, 2y)$ **3.** $(\frac{x}{\sqrt{x^2+y^2+4}}, \frac{y}{\sqrt{x^2+y^2+4}})$

5. $(1/x, 1/y)$

7. $(yz\cos(xyz), xz\cos(xyz), xy\cos(xyz))$

9. $(2x, 2y, 2z)$ **11.** $2\sqrt{2}$ **13.** $\frac{1}{\sqrt{3}}$

15. $\sqrt{3}\cos(1)$ **17.** increase: $(45, 20)$,
decrease: $(-45, -20)$

Section 2.5 (p. § 2.5)

1. local min. $(1, 0)$; saddle pt. $(-1, 0)$

3. local min. $(1, 1)$; local max. $(-1, -1)$; saddle pts. $(1, -1), (-1, 1)$ **5.** local min. $(1, -1)$; saddle pt. $(0, 0)$ **7.** local min. $(0, 0)$

9. local min. $(-1, 1/2)$ **11.** width = height = depth=10 **13.** $x = y = 4$, $z = 2$

Section 2.6 (p. § 2.6)

2. $(x_0, y_0) = (0, 0) : \to (0.2858, -0.3998)$; $(x_0, y_0) = (1, 1) : \to (1.03256, -1.94037)$

Section 2.7 (p. § 2.7)

1. min. $\left(\frac{-4}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$; max. $\left(\frac{4}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

3. min. $\left(\frac{20}{\sqrt{13}}, \frac{30}{\sqrt{13}}\right)$; max. $\left(-\frac{20}{\sqrt{13}}, -\frac{30}{\sqrt{13}}\right)$

4. min. $\left(\frac{-9}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$; max. $\left(\frac{9}{8}, \frac{\sqrt{59}}{4}, \frac{-1}{4}\right)$

5. $\frac{8abc}{3\sqrt{3}}$

Chapter 3

Section 3.1 (p. § 3.1)

1. 1 **3.** $\frac{7}{12}$ **5.** $\frac{7}{6}$ **7.** 5 **9.** $\frac{1}{2}$ **11.** 15

Section 3.2 (p. § 3.2)

1. 1 **3.** $8\ln 2 - 3$ **5.** $\frac{\pi}{4}$ **6.** $\frac{1}{4}$ **7.** 2 **9.** $\frac{1}{6}$ **10.** $\frac{6}{5}$

Section 3.3 (p. § 3.3)

1. $\frac{9}{2}$ **3.** $(2\cos(\pi^2) + \pi^4 - 2)/4$ **5.** $\frac{1}{6}$ **7.** 6

10. $\frac{1}{3}$

Section 3.4 (p. § 3.4)

1. The values should converge to ≈ 1.318. (Hint: In Java the exponential function e^x can be obtained with `Math.exp(x)`. Other languages have similar functions, otherwise use $e = 2.7182818284590455$ in your program.)
2. ≈ 1.146 **3.** ≈ 0.705 **4.** ≈ 0.168

Section 3.5 (p. § 3.5)

1. 8π **3.** $\frac{4\pi}{3}(8-3^{3/2})$ **7.** $1-\frac{\sin 2}{2}$ **9.** $2\pi ab$

Section 3.6 (p. § 3.6)

1. $(1, 8/3)$ **3.** $(0, \frac{4a}{3\pi})$ **5.** $(0, 3\pi/16)$
7. $(0, 0, 5a/12)$ **9.** $(7/12, 7/12, 7/12)$

Section 3.7 (p. § 3.7)

1. $\sqrt{\pi}$ **2.** 1 **6.** Both are $\frac{n}{(n+1)^2(n+2)}$ **7.** $\frac{1}{n}$

Chapter 4

Section 4.1 (p. § 4.1)

1. $1/2$ **3.** 23 **5.** 24π **7.** -2π **9.** 2π
11. 0

Section 4.2 (p. § 4.2)

1. 0 **3.** No **4.** Yes. $F(x,y) = \frac{x^2}{2} - \frac{y^2}{2}$
5. No **9.** (b) No. Hint: Think of how F is defined. **10.** Yes. $F(x,y) = axy + bx + cy + d$

Section 4.3 (p. § 4.3)

1. $16/15$ **3.** -5π **5.** Yes. $F(x,y) = xy^2 + x^3$ **7.** Yes. $F(x,y) = 4x^2y + 2y^2 + 3x$

Section 4.4 (p. § 4.4)

1. 216π **2.** 3 **3.** $12\pi/5$ **7.** $15/4$

Section 4.5 (p. § 4.5)

1. $2\sqrt{2}\,\pi^2$ **2.** $(17\sqrt{17}-5\sqrt{5})/3$ **3.** $2/5$

4. 2 **5.** $2\pi(\pi-1)$ **7.** $67/15$ **9.** 6

11. Yes **13.** No **19.** Hint: Think of how a vector field $\mathbf{f}(x,y) = P(x,y)\,\mathbf{i} + Q(x,y)\,\mathbf{j}$ in $\mathbb{R}^2$ can be extended in a natural way to be a vector field in $\mathbb{R}^3$.

Section 4.6 (p. § 4.6)

1. 0 **3.** $12\sqrt{x^2+y^2+z^2}$ **5.** $6(x+y+z)$

7. 12ρ **8.** $(4\rho^2-6)e^{-\rho^2}$ **9.** $-\frac{2z}{r^3}\,\mathbf{e}_r + \frac{1}{r^2}\,\mathbf{e}_z$

11. $\text{div}\ \mathbf{f} = \frac{2}{\rho} - \frac{\sin\theta}{\sin\phi} + \cot\phi$;

$\text{curl}\ \mathbf{f} = \cot\phi\,\cos\theta\,\mathbf{e}_\rho + 2\mathbf{e}_\theta - 2\cos\theta\,\mathbf{e}_\phi$

25. Hint: Start by showing that $\mathbf{e}_r = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$, $\mathbf{e}_\theta = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$, $\mathbf{e}_z = \mathbf{k}$.