8 Optical Fiber

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8.1 Optical Fiber: Method of Operation

In its simplest form, optical fiber consists of concentric regions of dielectric material as shown in Figure 8.1.

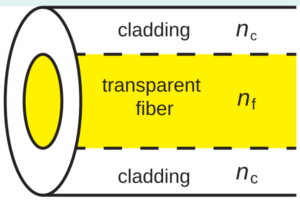

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Figure 8.1. *Construction of the simplest form of optical fiber.*

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A cross-section through the fiber reveals a circular region of transparent dielectric material through which light propagates. This is surrounded by a jacket of dielectric material commonly referred to as *cladding*.

A characteristic of the design of any optical fiber is that the permittivity of the fiber is greater than the permittivity of the cladding. As explained in Section 5.11, this creates conditions necessary for total internal reflection. The mechanism of total internal reflection contains the light within the fiber.

In the discipline of optics, the permittivity of a material is commonly quantified in terms of its *index of refraction*. Index of refraction is the square root of relative permittivity, and is usually assigned the symbol n. Thus, if we define the relative permittivities $\epsilon_{r,f} \triangleq \epsilon_f/\epsilon_0$ for the fiber and $\epsilon_{r,c} \triangleq \epsilon_c/\epsilon_0$ for the cladding, then

$$n_f \triangleq \sqrt{\epsilon_{r,f}} \tag{8.1}$$
$$n_c \triangleq \sqrt{\epsilon_{r,c}} \tag{8.2}$$

and

$$n_f > n_c \tag{8.3}$$

Figure 8.2 illustrates total internal reflection in an optical fiber. In this case, a ray of light in the fiber is incident on the boundary with the cladding. We may treat the light ray as a uniform plane wave. To see why, consider that optical wavelengths range from 120 nm to 700 nm in free space. Wavelength is slightly shorter than this in fiber; specifically, by a factor equal to the square root of the relative permittivity. The fiber is on the order of millimeters in diameter, which is about 4 orders of magnitude greater than the wavelength. Thus, from the perspective of the light ray, the fiber appears to be an unbounded half-space sharing a planar boundary with the cladding, which also appears to be an unbounded half-space.

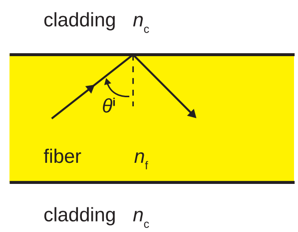

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Figure 8.2. *Total internal reflection in optical fiber.*

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Continuing under this presumption, the criterion for total internal reflection is (from Section 5.11):

$$\theta^i \geq \arcsin\sqrt{\frac{\epsilon_{r,c}}{\epsilon_{r,f}}} = \arcsin\frac{n_c}{n_f} \tag{8.4}$$

As long as rays of light approach from angles that satisfy this criterion, the associated power remains in the fiber, and is reflected onward. Otherwise, power is lost into the cladding.

Example

Example 8.1

Critical angle for optical fiber.

Typical values of n_f and n_c for an optical fiber are 1.52 and 1.49, respectively. What internal angle of incidence is required to maintain total internal reflection?

Solution. Using Equation 8.4, θ^i must be greater than about 78.8°. In practice this is quite reasonable since we desire light to be traveling approximately parallel the axis of the fiber (θ^i $\approx 90^\circ$) anyway.

The total internal reflection criterion imposes a limit on the radius of curvature of fiber optic cable. If fiber optic cable is bent such that the radius of curvature is too small, the critical angle will be exceeded at the bend. This will occur even for light rays which are traveling perfectly parallel to the axis of the fiber before they arrive at the bend.

Note that the cladding serves at least two roles. First, it determines the critical angle for total internal reflection, and subsequently determines the minimum radius of curvature for lossless operation of the fiber. Second, it provides a place for the evanescent surface waves associated with total internal reflection (Section 5.12) to exist without interference from objects in contact with the fiber.

Additional Reading:

- "Optical fiber (https://en.wikipedia.org/wiki/Optical_fiber)" on Wikipedia.

8.2 Acceptance Angle

In this section, we consider the problem of injecting light into a fiber optic cable. The problem is illustrated in Figure 8.3.

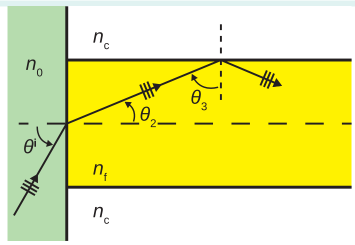

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Figure 8.3. *Injecting light into a fiber optic cable.*

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In this figure, we see light incident from a medium having index of refraction n_0, with angle of incidence θ^i. The light is transmitted with angle of transmission θ_2 into the fiber, and is subsequently incident on the surface of the cladding with angle of incidence θ_3. For light to propagate without loss within the cable, it is required that

$$\sin\theta_3 \geq \frac{n_c}{n_f} \tag{8.5}$$

since this criterion must be met in order for total internal reflection to occur.

Now consider the constraint that Equation 8.5 imposes on θ^i. First, we note that θ_3 is related to θ_2 as follows:

$$\theta_3 = \frac{\pi}{2} - \theta_2 \tag{8.6}$$

therefore

$$\begin{aligned} \sin\theta_3 &= \sin\left(\frac{\pi}{2} - \theta_2\right) && (8.7) \\ &= \cos\theta_2 && (8.8) \end{aligned}$$

so

$$\cos\theta_2 \geq \frac{n_c}{n_f} \tag{8.9}$$

Squaring both sides, we find:

$$\cos^2\theta_2 \geq \frac{n_c^2}{n_f^2} \tag{8.10}$$

Now invoking a trigonometric identity:

$$1 - \sin^2\theta_2 \geq \frac{n_c^2}{n_f^2} \tag{8.11}$$

so:

$$\sin^2\theta_2 \leq 1 - \frac{n_c^2}{n_f^2} \tag{8.12}$$

Now we relate the θ_2 to θ^i using Snell's law:

$$\sin\theta_2 = \frac{n_0}{n_f}\sin\theta^i \tag{8.13}$$

so Equation 8.12 may be written:

$$\frac{n_0^2}{n_f^2}\sin^2\theta^i \leq 1 - \frac{n_c^2}{n_f^2} \tag{8.14}$$

Now solving for $\sin\theta^i$, we obtain:

$$\sin\theta^i \leq \frac{1}{n_0}\sqrt{n_f^2 - n_c^2} \tag{8.15}$$

This result indicates the range of angles of incidence which result in total internal reflection within the fiber. The maximum value of θ^i which satisfies this condition is known as the *acceptance angle* θ_a, so:

$$\theta_a \triangleq \arcsin\left(\frac{1}{n_0}\sqrt{n_f^2 - n_c^2}\right) \tag{8.16}$$

This leads to the following insight:

In order to effectively launch light in the fiber, it is necessary for the light to arrive from within a cone having half-angle θ_a with respect to the axis of the fiber.

The associated *cone of acceptance* is illustrated in Figure 8.4.

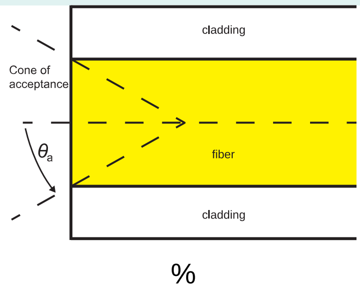

Figure 8.4. *Cone of acceptance.*

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It is also common to define the quantity *numerical aperture* NA as follows:

$$\mathrm{NA} \triangleq \frac{1}{n_0}\sqrt{n_f^2 - n_c^2} \tag{8.17}$$

Note that n_0 is typically very close to 1 (corresponding to incidence from air), so it is common to see NA defined as simply $\sqrt{n_f^2 - n_c^2}$. This parameter is commonly used in lieu of the acceptance angle in datasheets for fiber optic cable.

Example

Example 8.2

Acceptance angle.

Typical values of n_f and n_c for an optical fiber are 1.52 and 1.49, respectively. What are the numerical aperture and the acceptance angle?

Solution. Using Equation 8.17 and presuming $n_0 = 1$, we find NA $\cong \underline{0.30}$. Since $\sin\theta_a =$ NA, we find $\theta_a = \underline{17.5^\circ}$. Light must arrive from within 17.5° from the axis of the fiber in order to ensure total internal reflection within the fiber.

Additional Reading:

- "Optical fiber (https://en.wikipedia.org/wiki/Optical_fiber)" on Wikipedia.
- "Numerical aperture (https://en.wikipedia.org/wiki/Numerical_aperture)" on Wikipedia.

8.3 Dispersion in Optical Fiber

Light may follow a variety of paths through a fiber optic cable. Each of the paths has a different length, leading to a phenomenon known as *dispersion*. Dispersion distorts signals and limits the data rate of digital signals sent over fiber optic cable. In this section, we analyze this dispersion and its effect on digital signals.

Figure 8.5 shows the variety of paths that light may take through a straight fiber optic cable.

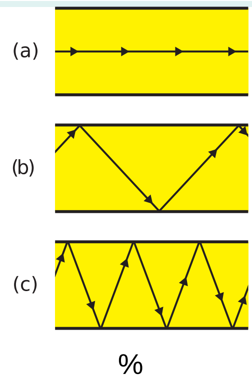

Figure 8.5. *Paths that light may take through a straight fiber optic cable: (a) Along axis, corresponding to minimum path length; (b) Intermediate between (a) and (c); (c) Maximum length, corresponding to threshold angle for total internal reflection.*

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The nominal path is shown in Figure 8.5(a), which is parallel to the axis of the cable. This path has the shortest associated propagation time. The path with the longest associated propagation time is shown in Figure 8.5(c). In this case, light bounces within the fiber, each time approaching the core-clad interface at the critical angle for total internal reflection. Any ray approaching at a greater angle is not completely reflected, and so would likely not survive to the end of the cable. Figure 8.5(b) represents the continuum of possibilities between the extreme cases of (a) and (c), with associated propagation times greater than that of case (a) but less than that of case (c).

Regardless of how light is inserted into the fiber, all possible paths depicted in Figure 8.5 are likely to exist. This is because fiber is rarely installed in a straight line, but rather follows a multiply-curved path. Each curve results in new angles of incidence upon the core-cladding boundary. The existence of these paths leads to dispersion. To see this, consider the input signal shown in Figure 8.6.

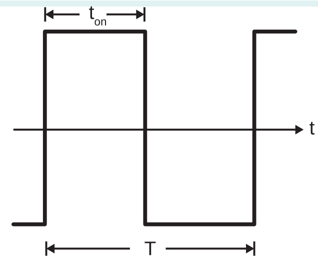

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Figure 8.6. *A digital signal that might be applied to the input of a fiber optic cable.*

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This signal has period T, during which time a pulse of length $t_{on} < T$ may be present. Let the *minimum* propagation time through the fiber (as in Figure 8.5(a)) be τ_{min}. Let the *maximum* propagation time through the fiber (as in Figure 8.5(c)) be τ_{max}. If $\tau_{max} - \tau_{min} \ll t_{on}$, we see little degradation in the signal output from the fiber. Otherwise, we observe a "smearing" of pulses at the output of the fiber, as shown in Figure 8.7.

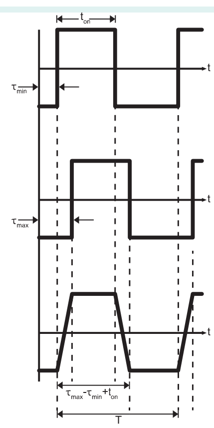

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Figure 8.7. *The effect of dispersion on the signal output from the fiber optic cable: Top: Arriving waveform corresponding to minimum path length, Middle: Arriving waveform corresponding to maximum path length, and Bottom: Sum of all arriving waveforms.*

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Any smearing may pose problems for whatever device is detecting pulses at the receiving end. However, as $\tau_{max} - \tau_{min}$ becomes a larger fraction of t_{on}, we see that it becomes possible for adjacent pulses to overlap. This presents a much more serious challenge in detecting individual pulses, so let us examine the overlap problem in greater detail. To avoid overlap:

$$\tau_{max} - \tau_{min} + t_{on} < T \tag{8.18}$$

Let us define the quantity $\tau \triangleq \tau_{max} - \tau_{min}$. This is sometimes referred to as the *delay spread*.[1] Thus, we obtain the following requirement for overlap-free transmission:

$$\tau < T - t_{on} \tag{8.19}$$

Note that the delay spread imposes a minimum value on T, which in turn imposes a maximum value on the rate at which information can be transmitted on the cable. So, we are motivated to calculate delay spread. The minimum propagation time t_{min} is simply the length l of the cable divided by the phase velocity v_p of the light within the core; i.e., $t_{min} = l/v_p$. Recall phase velocity is simply the speed of light in free space c divided by $\sqrt{\epsilon_r}$ where ϵ_r is the relative permittivity of the core material. Thus:

$$t_{min} = \frac{l}{v_p} = \frac{l}{c/\sqrt{\epsilon_r}} = \frac{l}{c/n_f} = \frac{ln_f}{c} \tag{8.20}$$

The maximum propagation time t_{max} is different because the maximum path length is different. Specifically, the maximum path length is not l, but rather $l/\cos\theta_2$ where θ_2 is the angle between the axis and the direction of travel as light crosses the axis. So, for example, $\theta_2 = 0$ for t_{min} since light in that case travels along the axis. The value of θ_2 for t_{max} is determined by the threshold angle for total internal reflection. This was determined in Section 8.2 to be given by:

$$\cos\theta_2 = \frac{n_c}{n_f} \qquad \text{(threshold value)} \tag{8.21}$$

so we obtain the following relationship:

$$t_{max} = \frac{l/\cos\theta_2}{v_p} = \frac{ln_f/n_c}{c/n_f} = \frac{ln_f^2}{cn_c} \tag{8.22}$$

and subsequently we find:

$$\boxed{\tau = l\frac{n_f}{c}\left(\frac{n_f}{n_c} - 1\right)} \tag{8.23}$$

Note that τ increases linearly with l. Therefore, the rate at which pulses can be sent without overlap decreases linearly with increasing length. In practical applications, this means that the maximum supportable data rate decreases as the length of the cable increases. This is true independently of media loss within the cable (which we have not yet even considered!). Rather, this is a fundamental limitation resulting from dispersion.

Example

Example 8.3

Maximum supportable data rate in multimode fiber optic cable.

A multimode fiber optic cable of length 1 m is used to transmit data using the scheme shown in Figure 8.6, with $t_{on} = T/2$. Values of n_f and n_c for this fiber are 1.52 and 1.49, respectively. What is the maximum supportable data rate?

Solution. Using Equation 8.23, we find the delay spread $\tau \cong 102$ ps. To avoid overlap, $T > 2\tau$; therefore, T greater than about 204 ps is required. The modulation scheme allows one bit per period, so the maximum data rate is $1/T \cong 4.9 \times 10^9$ bits per second; i.e., $\cong$ 4.9 Gb/s.

The finding of 4.9 Gb/s may seem like a pretty high data rate; however, consider what happens if the length increases to 1 km. It is apparent from Equation 8.23 that the delay spread will increase by a factor of 1000, so the maximum supportable data rate decreases by a factor of 1000 to a scant 4.9 Mb/s. Again, this is independent of any media loss within the fiber. To restore the higher data rate over this longer path, the dispersion must be reduced. One way to do this is to divide the link into smaller links separated by repeaters which can receive the dispersed signal, demodulate it, regenerate the original signal, and transmit the restored signal to the next repeater. Alternatively, one may employ “single mode” fiber, which has intrinsically less dispersion than the multimode fiber presumed in our analysis.

Additional Reading:

- “Optical fiber (https://en.wikipedia.org/wiki/Optical_fiber)” on Wikipedia.

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1. Full disclosure: There are many ways to define “delay spread,” but this definition is not uncommon and is useful in the present analysis. ↩

End Matter

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