1 Preliminary Concepts

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1.1 Units

The term “unit” refers to the measure used to express a physical quantity. For example, the mean radius of the Earth is about 6,371,000 meters; in this case, the unit is the meter.

A number like “6,371,000” becomes a bit cumbersome to write, so it is common to use a prefix to modify the unit. For example, the radius of the Earth is more commonly said to be 6371 kilometers, where one kilometer is understood to mean 1000 meters. It is common practice to use prefixes, such as “kilo-,” that yield values in the range of 0.001 to $10,000$. A list of standard prefixes appears in Table 1.1.

Table 1.1. *Prefixes used to modify units.*

Prefix	Abbreviation	Multiply by:
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Writing out the names of units can also become tedious. For this reason, it is common to use standard abbreviations; e.g., “6731 km” as opposed to “6371 kilometers,” where “k” is the standard abbreviation for the prefix “kilo” and “m” is the standard abbreviation for “meter.” A list of commonly-used base units and their abbreviations are shown in Table 1.2.

Table 1.2. *Some units that are commonly used in electromagnetics.*

Unit	Abbreviation	Quantifies:
ampere	A	electric current
coulomb	C	electric charge
farad	F	capacitance
henry	H	inductance
hertz	Hz	frequency
joule	J	energy
meter	m	distance
newton	N	force
ohm	Ω	resistance
second	s	time
tesla	T	magnetic flux density
volt	V	electric potential
watt	W	power
weber	Wb	magnetic flux

To avoid ambiguity, it is important to always indicate the units of a quantity; e.g., writing “6371 km” as opposed to “6371.” Failure to do so is a common source of error and misunderstandings. An example is the expression:

$$l = 3t$$

where l is length and t is time. It could be that l is in meters and t is in seconds, in which case “3” really means “3 m/s.” However, if it is intended that l is in kilometers and t is in hours, then “3” really means “3 km/h,” and the equation is literally different. To patch this up, one might write “$l = 3t$ m/s”; however, note that this does not resolve the ambiguity we just identified – i.e., we still don’t know the units of the constant “3.” Alternatively, one might write “$l = 3t$ where l is in meters and t is in seconds,” which is unambiguous but becomes quite awkward for more complicated expressions. A better solution is to write instead:

$$l = (3 \text{ m/s})\, t$$

or even better:

$$l = at \quad \text{where } a = 3 \text{ m/s}$$

since this separates the issue of units from the perhaps more-important fact that l is proportional to t and the constant of proportionality (a) is known.

The meter is the fundamental unit of length in the International System of Units, known by its French acronym "SI" and sometimes informally referred to as the "metric system."

In this work, we will use SI units exclusively.

Although SI is probably the most popular for engineering use overall, other systems remain in common use. For example, the English system, where the radius of the Earth might alternatively be said to be about 3959 miles, continues to be used in various applications and to a lesser or greater extent in various regions of the world. An alternative system in common use in physics and material science applications is the CGS ("centimeter-gram-second") system. The CGS system is similar to SI, but with some significant differences. For example, the base unit of energy in the CGS system is not the "joule" but rather the "erg," and the values of some physical constants become unitless. Therefore – once again – it is very important to include units whenever values are stated.

SI defines seven fundamental units from which all other units can be derived. These fundamental units are distance in meters (m), time in seconds (s), current in amperes (A), mass in kilograms (kg), temperature in kelvin (K), particle count in moles (mol), and luminosity in candela (cd). SI units for electromagnetic quantities such as coulombs (C) for charge and volts (V) for electric potential are derived from these fundamental units.

A frequently-overlooked feature of units is their ability to assist in error-checking mathematical expressions. For example, the electric field intensity may be specified in volts per meter (V/m), so an expression for the electric field intensity that yields units of V/m is said to be "dimensionally correct" (but not necessarily correct), whereas an expression that cannot be reduced to units of V/m *cannot* be correct.

Additional Reading:

- "International System of Units (https://en.wikipedia.org/wiki/International_System_of_Units)" on Wikipedia.
- "Centimetre-gram-second system of units (https://en.wikipedia.org/wiki/Centimetre-gram-second_system_of_units)" on Wikipedia.

1.2 Notation

The list below describes notation used in this book.

- *Vectors*: Boldface is used to indicate a vector; e.g., the electric field intensity vector will typically appear as $\mathbf{E}$. Quantities not in boldface are scalars. When writing by hand, it is common to write "$\overline{E}$" or "$\overrightarrow{E}$" in lieu of "$\mathbf{E}$."
- *Unit vectors*: A circumflex is used to indicate a unit vector; i.e., a vector having magnitude equal to one. For example, the unit vector pointing in the $+x$ direction will be indicated as $\hat{\mathbf{x}}$. In discussion, the quantity "$\hat{\mathbf{x}}$" is typically spoken "x hat."
- *Time*: The symbol t is used to indicate time.
- *Position*: The symbols (x, y, z), (ρ, ϕ, z), and (r, θ, ϕ) indicate positions using the Cartesian, cylindrical, and spherical coordinate systems, respectively. It is sometimes convenient to express position in a manner which is independent of a coordinate system; in this case, we typically use the symbol $\mathbf{r}$. For example, $\mathbf{r} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$ in the Cartesian coordinate system.
- *Phasors:* A tilde is used to indicate a phasor quantity; e.g., a voltage phasor might be indicated as $\widetilde{V}$, and the phasor representation of $\mathbf{E}$ will be indicated as $\widetilde{\mathbf{E}}$.
- *Curves, surfaces, and volumes:* These geometrical entities will usually be indicated in script; e.g., an open surface might be indicated as $\mathcal{S}$ and the curve bounding this surface might be indicated as $\mathcal{C}$. Similarly, the volume enclosed by a closed surface $\mathcal{S}$ may be indicated as $\mathcal{V}$.
- *Integrations over curves, surfaces, and volumes* will usually be indicated using a single integral sign with the appropriate subscript. For example:

$$\int_{\mathcal{C}} \cdots \, dl \quad \text{is an integral over the curve } \mathcal{C}$$

$$\int_{\mathcal{S}} \cdots \, ds \quad \text{is an integral over the surface } \mathcal{S}$$

$$\int_{\mathcal{V}} \cdots \, dv \quad \text{is an integral over the volume } \mathcal{V}.$$

- *Integrations over* *closed* *curves and surfaces* will be indicated using a circle superimposed on the integral sign. For example:

$$\oint_{\mathcal{C}} \cdots \, dl \quad \text{is an integral over the closed curve } \mathcal{C}$$

$$\oint_{\mathcal{S}} \cdots \, ds \quad \text{is an integral over the closed surface } \mathcal{S}$$

  A "closed curve" is one which forms an unbroken loop; e.g., a circle. A "closed surface" is one which encloses a volume with no openings; e.g., a sphere.

- The symbol "$\cong$" means "approximately equal to." This symbol is used when equality exists, but is not being expressed with exact numerical precision. For example, the ratio of the circumference of a circle to its diameter is π, where $\pi \cong 3.14$.

- The symbol "$\approx$" also indicates "approximately equal to," but in this case the two quantities are unequal even if expressed with exact numerical precision. For example, $e^x = 1 + x + x^2/2 + ...$ as an infinite series, but $e^x \approx 1 + x$ for $x \ll 1$. Using this approximation, $e^{0.1} \approx 1.1$, which is in good agreement with the actual value $e^{0.1} \cong 1.1052$.

- The symbol "$\sim$" indicates "on the order of," which is a relatively weak statement of equality indicating that the indicated quantity is within a factor of 10 or so of the indicated value. For example, $\mu \sim 10^5$ for a class of iron alloys, with exact values being larger or smaller by a factor of 5 or so.

- The symbol "$\triangleq$" means "is defined as" or "is equal as the result of a definition."

- *Complex numbers*: $j \triangleq \sqrt{-1}$.

- See Appendix C for notation used to identify commonly-used physical constants.

1.3 Coordinate Systems

The coordinate systems most commonly used in engineering analysis are the *Cartesian*, *cylindrical*, and *spherical* systems. These systems are illustrated in Figures 1.1, 1.2, and 1.3, respectively. Note that the use of variables is not universal; in particular, it is common to encounter the use of r in lieu of ρ for the radial coordinate in the cylindrical system, and the use of R in lieu of r for the radial coordinate in the spherical system.

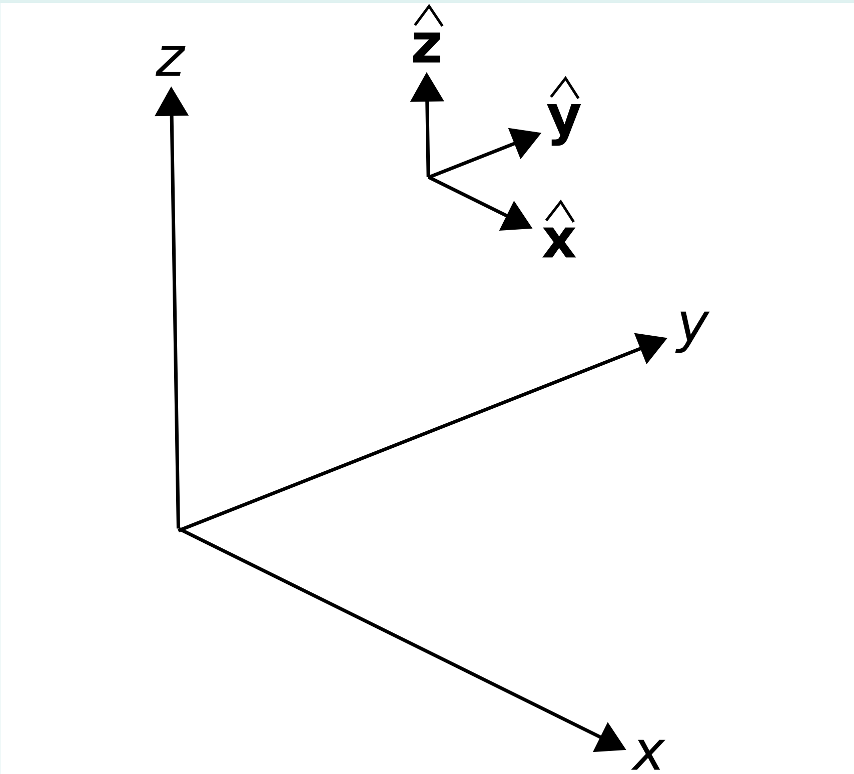

Figure 1.1. *Cartesian coordinate system.*

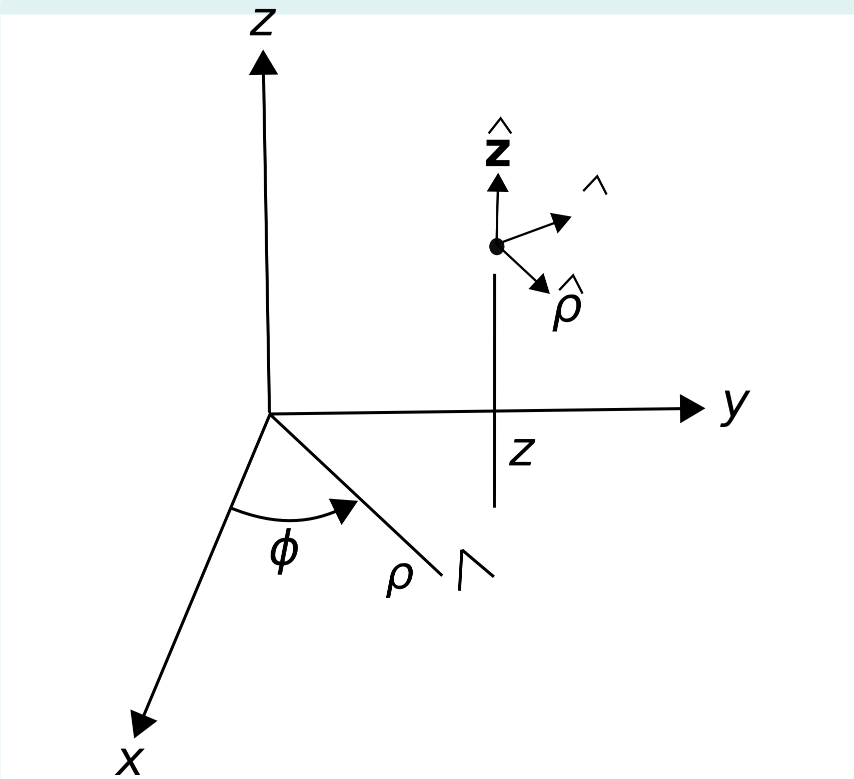

Figure 1.2. *Cylindrical coordinate system.*

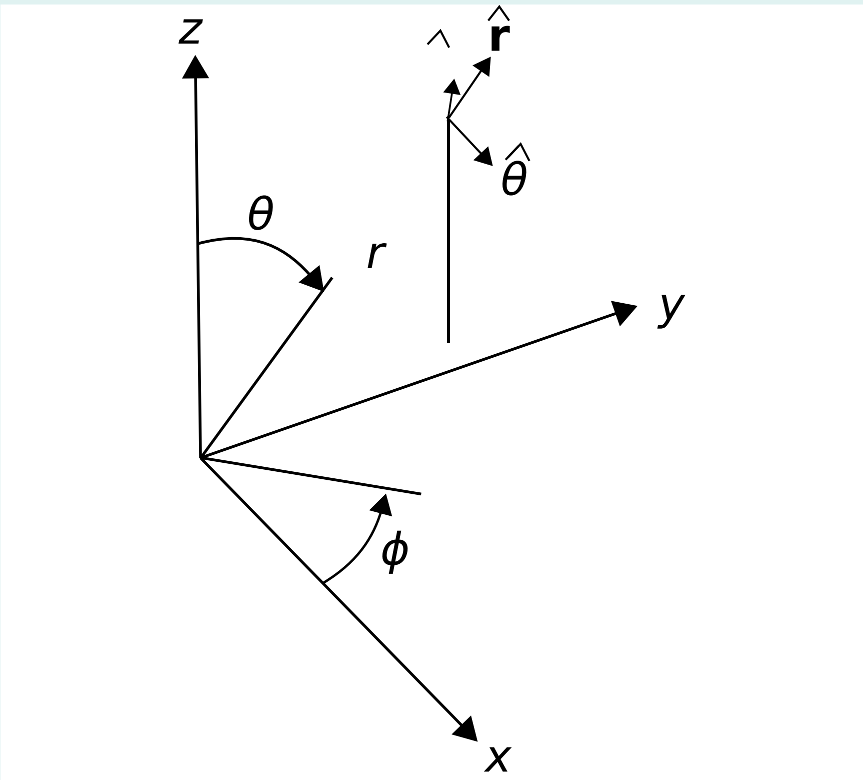

Figure 1.3. *Spherical coordinate system.*

Additional Reading:

- "Cylindrical coordinate system (https://en.wikipedia.org/wiki/Cylindrical_coordinate_system)" on Wikipedia.
- "Spherical coordinate system (https://en.wikipedia.org/wiki/Spherical_coordinate_system)" on Wikipedia.

1.4 Electromagnetic Field Theory: A Review

This book is the second in a series of textbooks on electromagnetics. This section presents a summary of electromagnetic field theory concepts presented in the previous volume.

Electric charge and current. Charge is the ultimate source of the electric field and has SI base units of coulomb (C). An important source of charge is the electron, whose charge is defined to be negative. However, the term "charge" generally refers to a large number of charge carriers of various types, and whose relative net charge may be either positive or negative. Distributions of charge may alternatively be expressed in terms of line charge density ρ_l (C/m), surface charge density ρ_s (C/m^2), or volume charge density ρ_v (C/m^3). Electric current describes the net motion of charge. Current is expressed in SI base units of amperes (A) and may alternatively be quantified in terms of surface current density $\mathbf{J}_s$ (A/m) or volume current density $\mathbf{J}$ (A/m^2).

Electrostatics. Electrostatics is the theory of the electric field subject to the constraint that charge does not accelerate. That is, charges may be motionless ("static") or move without acceleration ("steady current").

The electric field may be interpreted in terms of energy or flux. The energy interpretation of the electric field is referred to as *electric field intensity* $\mathbf{E}$ (SI base units of N/C or V/m), and is related to the energy associated with charge and forces between charges. One finds that the electric potential (SI base units of V) over a path $\mathcal{C}$ is given by

$$V = -\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l} \tag{1.1}$$

The principle of *independence of path* means that only the endpoints of $\mathcal{C}$ in Equation 1.1, and no other details of $\mathcal{C}$, matter. This leads to the finding that the electrostatic field is *conservative*; i.e.,

$$\oint_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l} = 0 \tag{1.2}$$

This is referred to as *Kirchoff's voltage law for electrostatics*. The inverse of Equation 1.1 is

$$\mathbf{E} = -\nabla V \tag{1.3}$$

That is, the electric field intensity points in the direction in which the potential is most rapidly decreasing, and the magnitude is equal to the rate of change in that direction.

The flux interpretation of the electric field is referred to as *electric flux density* $\mathbf{D}$ (SI base units of C/m^2), and quantifies the effect of charge as a flow emanating from the charge. *Gauss' law for electric fields* states that the electric flux through a closed surface is equal to the enclosed charge Q_{encl}; i.e.,

$$\oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{s} = Q_{encl} \tag{1.4}$$

Within a material region, we find

$$\mathbf{D} = \epsilon \mathbf{E} \tag{1.5}$$

where ϵ is the *permittivity* (SI base units of F/m) of the material. In free space, ϵ is equal to

$$\epsilon_0 \triangleq 8.854 \times 10^{-12} \text{ F/m} \tag{1.6}$$

It is often convenient to quantify the permittivity of material in terms of the unitless *relative permittivity* $\epsilon_r \triangleq \epsilon/\epsilon_0$.

Both $\mathbf{E}$ and $\mathbf{D}$ are useful as they lead to distinct and independent boundary conditions at the boundary between dissimilar material regions. Let us refer to these regions as Regions 1 and 2, having fields $(\mathbf{E}_1, \mathbf{D}_1)$ and $(\mathbf{E}_2, \mathbf{D}_2)$, respectively. Given a vector $\hat{\mathbf{n}}$ perpendicular to the boundary and pointing into Region 1, we find

$$\hat{\mathbf{n}} \times [\mathbf{E}_1 - \mathbf{E}_2] = 0 \tag{1.7}$$

i.e., the tangential component of the electric field is continuous across a boundary, and

$$\hat{\mathbf{n}} \cdot [\mathbf{D}_1 - \mathbf{D}_2] = \rho_s \tag{1.8}$$

i.e., any discontinuity in the normal component of the electric field must be supported by a surface charge distribution on the boundary.

Magnetostatics. Magnetostatics is the theory of the magnetic field in response to steady current or the intrinsic magnetization of materials. Intrinsic magnetization is a property of some materials, including permanent magnets and magnetizable materials.

Like the electric field, the magnetic field may be quantified in terms of energy or flux. The flux interpretation of the magnetic field is referred to as *magnetic flux density* $\mathbf{B}$ (SI base units of Wb/m^2), and quantifies the field as a flow associated with, but not emanating from, the source of the field. The magnetic flux Φ (SI base units of Wb) is this flow measured through a specified surface. *Gauss' law for magnetic fields* states that

$$\oint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = 0 \tag{1.9}$$

i.e., the magnetic flux through a closed surface is zero. Comparison to Equation 1.4 leads to the conclusion that the source of the magnetic field cannot be localized; i.e., there is no “magnetic charge” analogous to electric charge. Equation 1.9 also leads to the conclusion that magnetic field lines form closed loops.

The energy interpretation of the magnetic field is referred to as *magnetic field intensity* $\mathbf{H}$ (SI base units of A/m), and is related to the energy associated with sources of the magnetic field. *Ampere's law for magnetostatics* states that

$$\oint_{\mathcal{C}} \mathbf{H} \cdot d\mathbf{l} = I_{encl} \tag{1.10}$$

where I_{encl} is the current flowing past any open surface bounded by $\mathcal{C}$.

Within a homogeneous material region, we find

$$\mathbf{B} = \mu \mathbf{H} \tag{1.11}$$

where μ is the *permeability* (SI base units of H/m) of the material. In free space, μ is equal to

$$\mu_0 \triangleq 4\pi \times 10^{-7} \text{ H/m}. \tag{1.12}$$

It is often convenient to quantify the permeability of material in terms of the unitless *relative permeability* $\mu_r \triangleq \mu/\mu_0$.

Both $\mathbf{B}$ and $\mathbf{H}$ are useful as they lead to distinct and independent boundary conditions at the boundaries between dissimilar material regions. Let us refer to these regions as Regions 1 and 2, having fields $(\mathbf{B}_1, \mathbf{H}_1)$ and $(\mathbf{B}_2, \mathbf{H}_2)$, respectively. Given a vector $\hat{\mathbf{n}}$ perpendicular to the boundary and pointing into Region 1, we find

$$\hat{\mathbf{n}} \cdot [\mathbf{B}_1 - \mathbf{B}_2] = 0 \tag{1.13}$$

i.e., the normal component of the magnetic field is continuous across a boundary, and

$$\hat{\mathbf{n}} \times [\mathbf{H}_1 - \mathbf{H}_2] = \mathbf{J}_s \tag{1.14}$$

i.e., any discontinuity in the tangential component of the magnetic field must be supported by current on the boundary.

Maxwell's equations. Equations 1.2, 1.4, 1.9, and 1.10 are *Maxwell's equations* for static fields in integral form. As indicated in Table 1.3, these equations may alternatively be expressed in differential form. The principal advantage of the differential forms is that they apply at each point in space (as opposed to regions defined by $\mathcal{C}$ or $\mathcal{S}$), and subsequently can be combined with the boundary conditions to solve complex problems using standard methods from the theory of differential equations.

Table 1.3. *Comparison of Maxwell's equations for static and time-varying electromagnetic fields. Differences in the time-varying case relative to the static case are highlighted in blue.*

	Electrostatics /	**Time-Varying**
	Magnetostatics	**(Dynamic)**
Electric & magnetic	independent	possibly coupled
fields are...		
Maxwell's eqns.	$\oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$	$\oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$
(integral)	$\oint_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l} = 0$	Extension "color" failed to load
	$\oint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = 0$	$\oint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = 0$
	$\oint_{\mathcal{C}} \mathbf{H} \cdot d\mathbf{l} = I_{encl}$	$\oint_{\mathcal{C}} \mathbf{H} \cdot d\mathbf{l} = I_{encl}\backslash\text{color}blue + \int_{\mathcal{S}} \frac{\partial}{\partial t}\mathbf{D} \cdot d\mathbf{s}$
Maxwell's eqns.	$\nabla \cdot \mathbf{D} = \rho_v$	$\nabla \cdot \mathbf{D} = \rho_v$
(differential)	$\nabla \times \mathbf{E} = 0$	$\nabla \times \mathbf{E} = \backslash\text{color}blue - \frac{\partial}{\partial t}\mathbf{B}$
	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \times \mathbf{H} = \mathbf{J}\backslash\text{color}blue + \frac{\partial}{\partial t}\mathbf{D}$

Conductivity. Some materials consist of an abundance of electrons which are loosely-bound to the atoms and molecules comprising the material. The force exerted on these electrons by an electric field may be sufficient to overcome the binding force, resulting in motion of the associated charges and subsequently current. This effect is quantified by *Ohm's law for electromagnetics*:

$$\mathbf{J} = \sigma \mathbf{E} \tag{1.15}$$

where $\mathbf{J}$ in this case is the *conduction current* determined by the conductivity σ (SI base units of S/m). Conductivity is a property of a material that ranges from negligible (i.e., for "insulators") to very large for good conductors, which includes most metals.

A *perfect conductor* is a material within which $\mathbf{E}$ is essentially zero regardless of $\mathbf{J}$. For such material, $\sigma \to \infty$. Perfect conductors are said to be *equipotential regions*; that is, the potential difference between any two points within a perfect conductor is zero, as can be readily verified using Equation 1.1.

Time-varying fields. *Faraday's law* states that a time-varying magnetic flux induces an electric potential in a closed loop as follows:

$$V = -\frac{\partial}{\partial t}\Phi \tag{1.16}$$

Setting this equal to the left side of Equation 1.2 leads to the *Maxwell-Faraday equation* in integral form:

$$\oint_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} \tag{1.17}$$

where $\mathcal{C}$ is the closed path defined by the edge of the open surface $\mathcal{S}$. Thus, we see that a time-varying magnetic flux is able to generate an electric field. We also observe that electric and magnetic fields become coupled when the magnetic flux is time-varying.

An analogous finding leads to the general form of Ampere's law:

$$\oint_{\mathcal{C}} \mathbf{H} \cdot d\mathbf{l} = I_{encl} + \int_{\mathcal{S}} \frac{\partial}{\partial t} \mathbf{D} \cdot d\mathbf{s} \tag{1.18}$$

where the new term is referred to as *displacement current*. Through the displacement current, a time-varying electric flux may be a source of the magnetic field. In other words, we see that the electric and magnetic fields are coupled when the electric flux is time-varying.

Gauss' law for electric and magnetic fields, boundary conditions, and constitutive relationships (Equations 1.5, 1.11, and 1.15) are the same in the time-varying case.

As indicated in Table 1.3, the time-varying version of Maxwell's equations may also be expressed in differential form. The differential forms make clear that variations in the electric field with respect to position are associated with variations in the magnetic field with respect to time (the Maxwell-Faraday equation), and vice-versa (Ampere's law).

Time-harmonic waves in source-free and lossless media. The coupling between electric and magnetic fields in the time-varying case leads to wave phenomena. This is most easily analyzed for fields which vary sinusoidally, and may thereby be expressed as phasors.[1] Phasors, indicated in this book by the tilde ("$\sim$"), are complex-valued quantities representing the magnitude and phase of the associated sinusoidal waveform. Maxwell's equations in differential phasor form are:

$$\nabla \cdot \widetilde{\mathbf{D}} = \tilde{\rho}_v \tag{1.19}$$

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega \widetilde{\mathbf{B}} \tag{1.20}$$

$$\nabla \cdot \widetilde{\mathbf{B}} = 0 \tag{1.21}$$

$$\nabla \times \widetilde{\mathbf{H}} = \widetilde{\mathbf{J}} + j\omega \widetilde{\mathbf{D}} \tag{1.22}$$

where $\omega \triangleq 2\pi f$, and where f is frequency (SI base units of Hz). In regions which are free of sources (i.e., charges and currents) and consisting of loss-free media (i.e., $\sigma = 0$), these equations reduce to the following:

$$\nabla \cdot \widetilde{\mathbf{E}} = 0 \tag{1.23}$$
$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}} \tag{1.24}$$
$$\nabla \cdot \widetilde{\mathbf{H}} = 0 \tag{1.25}$$
$$\nabla \times \widetilde{\mathbf{H}} = +j\omega\epsilon\widetilde{\mathbf{E}} \tag{1.26}$$

where we have used the relationships $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ to eliminate the flux densities $\mathbf{D}$ and $\mathbf{B}$, which are now redundant. Solving Equations 1.23–1.26 for $\mathbf{E}$ and $\mathbf{H}$, we obtain the vector wave equations:

$$\nabla^2\widetilde{\mathbf{E}} + \beta^2\widetilde{\mathbf{E}} = 0 \tag{1.27}$$
$$\nabla^2\widetilde{\mathbf{H}} + \beta^2\widetilde{\mathbf{H}} = 0 \tag{1.28}$$

where

$$\beta \triangleq \omega\sqrt{\mu\epsilon} \tag{1.29}$$

Waves in source-free and lossless media are solutions to the vector wave equations.

Uniform plane waves in source-free and lossless media. An important subset of solutions to the vector wave equations are uniform plane waves. Uniform plane waves result when solutions are constrained to exhibit constant magnitude and phase in a plane. For example, if this plane is specified to be perpendicular to z (i.e., $\partial/\partial x = \partial/\partial y = 0$) then solutions for $\widetilde{\mathbf{E}}$ have the form:

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}\widetilde{E}_x + \hat{\mathbf{y}}\widetilde{E}_y \tag{1.30}$$

where

$$\widetilde{E}_x = E_{x0}^{+}e^{-j\beta z} + E_{x0}^{-}e^{+j\beta z} \tag{1.31}$$
$$\widetilde{E}_y = E_{y0}^{+}e^{-j\beta z} + E_{y0}^{-}e^{+j\beta z} \tag{1.32}$$

and where E_{x0}^{+}, E_{x0}^{-}, E_{y0}^{+}, and E_{y0}^{-} are constant complex-valued coefficients which depend on sources and boundary conditions. The first term and second terms of Equations 1.31 and 1.32 correspond to waves traveling in the $+\hat{\mathbf{z}}$ and $-\hat{\mathbf{z}}$ directions, respectively. Because $\widetilde{\mathbf{H}}$ is a solution to the same vector wave equation, the solution for $\mathbf{H}$ is identical except with different coefficients.

The scalar components of the plane waves described in Equations 1.31 and 1.32 exhibit the same characteristics as other types of waves, including sound waves and voltage and current waves in transmission lines. In particular, the phase velocity of waves propagating in the $+\hat{\mathbf{z}}$ and $-\hat{\mathbf{z}}$ direction is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} \tag{1.33}$$

and the wavelength is

$$\lambda = \frac{2\pi}{\beta} \tag{1.34}$$

By requiring solutions for $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{H}}$ to satisfy the Maxwell curl equations (i.e., the Maxwell-Faraday equation and Ampere's law), we find that $\widetilde{\mathbf{E}}$, $\widetilde{\mathbf{H}}$, and the direction of propagation $\hat{\mathbf{k}}$ are mutually perpendicular. In particular, we obtain the *plane wave relationships*:

$$\widetilde{\mathbf{E}} = -\eta\hat{\mathbf{k}} \times \widetilde{\mathbf{H}} \tag{1.35}$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{k}} \times \widetilde{\mathbf{E}} \tag{1.36}$$

where

$$\eta \triangleq \sqrt{\frac{\mu}{\epsilon}} \tag{1.37}$$

is the *wave impedance*, also known as the *intrinsic impedance of the medium*, and $\hat{\mathbf{k}}$ is in the same direction as $\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}$.

The power density associated with a plane wave is

$$S = \frac{\left|\widetilde{\mathbf{E}}\right|^2}{2\eta} \tag{1.38}$$

where S has SI base units of W/m^2, and here it is assumed that $\widetilde{\mathbf{E}}$ is in peak (as opposed to rms) units.

Commonly-assumed properties of materials. Finally, a reminder about commonly-assumed properties of the material constitutive parameters ϵ, μ, and σ. We often assume these parameters exhibit the following properties:

- *Homogeneity*. A material that is *homogeneous* is uniform over the space it occupies; that is, the values of its constitutive parameters are constant at all locations within the material.
- *Isotropy*. A material that is *isotropic* behaves in precisely the same way regardless of how it is oriented with respect to sources, fields, and other materials.
- *Linearity*. A material is said to be *linear* if its properties do not depend on the sources and fields applied to the material. Linear media exhibit *superposition*; that is, the response to multiple sources is equal to the sum of the responses to the sources individually.
- *Time-invariance*. A material is said to be *time-invariant* if its properties do not vary as a function of time.

Additional Reading:

- "Maxwell's Equations (https://en.wikipedia.org/wiki/Maxwell's_equations)" on Wikipedia.
- "Wave Equation (https://en.wikipedia.org/wiki/Wave_equation)" on Wikipedia.
- "Electromagnetic Wave Equation (https://en.wikipedia.org/wiki/Electromagnetic_wave_equation)" on Wikipedia.
- "Electromagnetic radiation (https://en.wikipedia.org/wiki/Electromagnetic_radiation)" on Wikipedia.

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1. Sinusoidally-varying fields are sometimes also said to be *time-harmonic*. ↩

End Matter

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